§ 5 (p. 249 ff.). Everywhere, both in the formulation of the theorem and in its proof, we replace the symbol '*Tr*' by the symbol '*Pr*' which denotes the class of all provable sentences of the theory under consideration and can be defined in the metatheory (cf. e.g. Def. 17 in § 2). In accordance with the first part of Th. I we can obtain the negation of one of the sentences in condition (α) of convention **T** of § 3 as a consequence of the definition of the symbol '*Pr*' (provided we replace '*Tr*' in this convention by '*Pr*'). In other words we can construct a sentence x of the science in question which satisfies the following condition:

it is not true that $x \in Pr$ *if and only if* p

or in equivalent formulation:

(1) $x \bar{\in} Pr$ *if and only if* p

where the symbol 'p' represents the whole sentence x (in fact we may choose the sentence $\bigcup_1^3(\iota_k \cdot \phi_k)$ constructed in the proof of Th. I as x).

We shall show that the sentence x is actually undecidable and at the same time true. For this purpose we shall pass to a metatheory of higher order; Th. I then obviously remains valid. According to Thesis A we can construct, on the basis of the enriched metatheory, a correct definition of truth concerning all the sentences of the theory studied. If we denote the class of all true sentences by the symbol '*Tr*' then—in accordance with convention **T**—the sentence x which we have constructed will satisfy the following condition:

(2) $x \in Tr$ *if and only if* p;

from (1) and (2) we obtain immediately

(3) $x \bar{\in} Pr$ *if and only if* $x \in Tr$.

Moreover, if we denote the negation of the sentence x by the symbol '$\bar{x}$' we can derive the following theorems from the definition of truth (cf. Ths. 1 and 5 in § 3):

(4) *either* $x \bar{\in} Tr$ *or* $\bar{x} \bar{\in} Tr$;

(5) *if* $x \in Pr$, *then* $x \in Tr$;

(6) *if* $\bar{x} \in Pr$, *then* $\bar{x} \in Tr$;

From (3) and (5) we infer without difficulty that

(7) $$x \in Tr$$

and that

(8) $$x \,\bar{\in}\, Pr.$$

In view of (4) and (7) we have $\bar{x} \,\bar{\in}\, Tr$, which together with (6) gives the formula

(9) $$\bar{x} \,\bar{\in}\, Pr.$$

The formulas (8) and (9) together express the fact that x is an undecidable sentence; moreover from (7) it follows that x is a true sentence.

By establishing the truth of the sentence x we have *eo ipso* —by reason of (2)—also proved x itself in the metatheory. Since, moreover, the metatheory can be interpreted in the theory enriched by variables of higher order (cf. p. 184) and since in this interpretation the sentence x, which contains no specific term of the metatheory, is its own correlate, the proof of the sentence x given in the metatheory can automatically be carried over into the theory itself: the sentence x which is undecidable in the original theory becomes a decidable sentence in the enriched theory.

I should like to draw attention here to an analogous result. For every deductive science in which arithmetic is contained it is possible to specify arithmetical notions which, so to speak, belong intuitively to this science, but which cannot be defined on the basis of this science. With the help of methods which are completely analogous to those used in the construction of the definition of truth, it is nevertheless possible to show that these concepts can be so defined provided the science is enriched by the introduction of variables of higher order.[1]

In conclusion it can be affirmed that the definition of truth and, more generally, the establishment of semantics enables us to match some important negative results which have been obtained

[1] Cf. my summary, 'Über definierbare Mengen reeller Zahlen,' *Annales de la Société Polonaise de Mathématique*, t. ix, année 1930, Kraków, 1931, pp. 206–7 (report on a lecture given on 16 December 1930 at the Lemberg Section of the Polish Mathematical Society); the ideas there sketched were in part developed later in VI. Cf. VI, p. 110, Bibliographical Note.