

Formalizing Self-Reference Paradox using Predicate Logic

We begin with the hypothetical assumption that Tarski's 1933 formula $\forall \text{True}(x) \varphi(x)$ has been defined such that $\forall x \text{Tarski:True}(x) \leftrightarrow \text{Boolean-True}$. On the basis of this logical premise we formalize the Truth Teller Paradox: "This sentence is true." showing syntactically how self-reference paradox is semantically ungrounded.

<https://plato.stanford.edu/entries/tarski-truth/#ForCor>

1.2 Formal correctness

The definition of *True* should be 'formally correct'. This means that it should be a sentence of the form: For all x , $\text{True}(x)$ if and only if $\varphi(x)$,

where *True* never occurs in φ ; or failing this, that the definition should be provably equivalent to a sentence of this form. The equivalence must be provable using axioms of the metalanguage that don't contain *True*. Definitions of the kind displayed above are usually called *explicit*, though Tarski in 1933 called them *normal*.

"This sentence is true." Formalized as this predicate logic:

$\exists x \in \text{Propositions} \exists P \in \text{Properties} \exists T \in \text{Predicates} | (x \leftrightarrow P(x)) \ \& \ (P(x) \leftrightarrow T(x))$

Simplified as this formula

$x \leftrightarrow \text{hasProperty}(x, \text{True}(x))$

When the above x is plugged into $\text{True}(x)$ to be evaluated we get

- (1) $\text{True}(\text{hasProperty}(x, \text{True}(x)))$
- (2) $\text{True}(\text{hasProperty}(x, \text{True}(\text{hasProperty}(x, \text{True}(x)))))$
- (3) $\text{True}(\text{hasProperty}(x, \text{True}(\text{hasProperty}(x, \text{True}(\text{hasProperty}(x, \text{True}(x)))))$
- (n) ... On and on to an infinitely recursive depth.

Copyright 2016, 2017 by Pete Olcott