

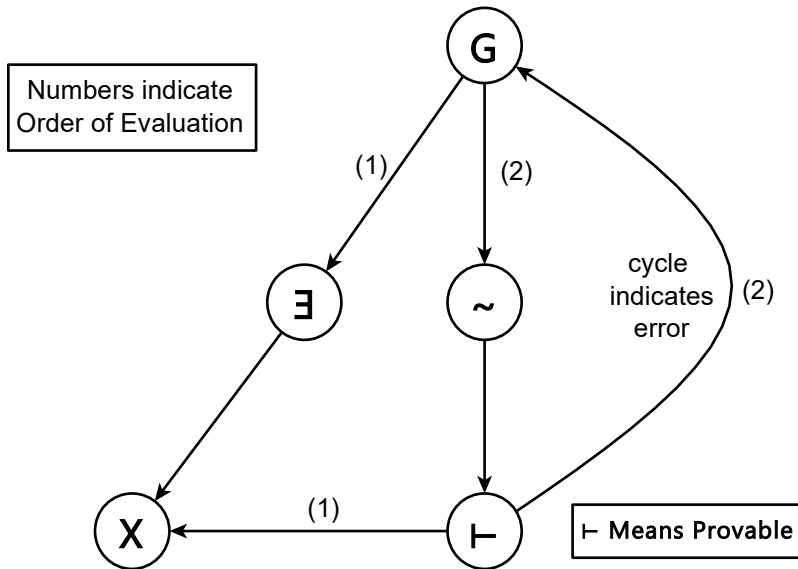
Evaluating the logical equivalent of the minimal essence of the Gödel sentence in Minimal Type Theory and Prolog

G := $\exists X \sim \text{Provable}(X, G)$ // Written in Minimal Type Theory **

Automatically translated into a Directed Acyclic Graph by the MTT compiler

```
[01] G           (02) (04)
[02] THERE_EXISTS (03)
[03] X
[04] NOT         (05)
[05] Provable   (03) (01) // cycle indicates
                        // infinite evaluation loop
```

** x := y means x is defined to be another name for y



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Finally, a note about how Prolog matching sometimes differs from the unification used in Resolution. Most Prolog systems will allow you to satisfy goals like:

```
equal(X, X).  
?- equal(foo(Y), Y).
```

that is, they will allow you to match a term against an uninstantiated subterm of itself. In this example, `foo(Y)` is matched against `Y`, which appears within it. As a result, `Y` will stand for `foo(Y)`, which is `foo(foo(Y))` (because of what `Y` stands for), which is `foo(foo(foo(Y)))`, and so on. So `Y` ends up standing for some kind of infinite structure.

Note that, whereas they may allow you to construct something like this, most Prolog systems will not be able to write it out at the end. According to the formal definition of Unification, this kind of “infinite term” should never come to exist. Thus Prolog systems that allow a term to match an uninstantiated subterm of itself do not act correctly as Resolution theorem provers. In order to make them do so, we would have to add a check that a variable cannot be instantiated to something containing itself. Such a check, an occurs check, would be straightforward to implement, but would slow down the execution of Prolog programs considerably. Since it would only affect very few programs, most implementors have simply left it out ¹.

¹ The Prolog standard states that the result is undefined if a Prolog system attempts to match a term against an uninstantiated subterm of itself, which means that programs which cause tills to happen will not be portable. A portable program should ensure that wherever an occurs check might be applicable the built-in predicate `unify_with_occurs_check/2` is used explicitly instead of the normal unification operation of the Prolog implementation. As its name suggests, this predicate acts like `=/2` except that it fails if an occurs check detects an illegal attempt to instantiate a variable. **END-OF-QUOTED-MATERIAL**