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Preface

This book is an investigation into the logic of truth. The investigation is provoked by the liar paradox, which shows that our naive understanding of truth, which is characterized by the acceptance of Tarski's schema

(T) $\ulcorner\phi\urcorner$ is true if and only if ϕ

is inconsistent. The aim of the investigation is to develop a new understanding of truth that does not fall prey to contradictions.

There are scarcely any philosophical problems of greater urgency than the liar paradox, for there are scarcely any concepts more central to our philosophical understanding than the concept of truth. The notions of truth and reference lie at the very center of all our attempts to understand how our language is linked to the world around us. These are the notions we need to use if we want to understand the astonishing fact that my utterance of the sentence 'The Yuan emperors ruled harshly' is somehow intimately connected with events that happened seven hundred years ago half a world away. The liar antinomy and the closely related antinomies involving reference show us, quite unmistakably, that our present way of thinking about truth and reference is inconsistent. Unless we can devise new ways of thinking about truth and reference which rise above the antinomies, we shall not have even the beginning of a satisfactory understanding of human language.

We want to replace our naive conception of truth by a scientific conception that serves the same purposes without falling prey to inconsistencies. The relation between our old and new conceptions of truth will be the same as the relation between our old, prescientific understanding of space and time and the understanding of space and time that we get from modern science.

Where do we begin? Schema (T) is so deeply embedded in our ordinary thinking about truth that we might fear that, once we decide to give (T) up, we should become so badly disoriented that we would not be able to talk about truth at all. A starting point is provided by some advice of Wittgenstein. In trying to understand a philosophically troublesome concept, do not focus all your attention upon how the concept behaves when it is on philosophical holiday. Pay attention to the everyday, unproblematic, nonphilosophical work the concept does.

When we look at the nonphilosophical work done by the concept of truth,

CHAPTER 4

Kripke and 3-valued Logic

Kripke [1975] has significantly advanced our understanding of the problems raised by the paradoxes by applying to these problems the methods of the mathematical theory of inductive definitions. Kripke develops an account according to which the paradoxical sentences are neither true nor false, utilizing the strong 3-valued logic of S.C. Kleene [1952, §54] to describe the logical properties of such non-truth-valued sentences. The construction Kripke develops is an extremely versatile mathematical tool that can be fruitfully used for a variety of philosophical purposes.

As Kripke emphasizes [1975, p. 77], his results do not depend crucially upon the choice of the Kleene 3-valued logic as the method for handling truth-value gaps. A variety of logics for languages with truth-value gaps have been proposed, and analogues to the results Kripke obtains using the 3-valued logic can be obtained for most of these other logics.[1] But whereas the mathematical results will be the same, their philosophical significance may vary. The discussion in this chapter is intended to apply only to the particular version of Kripke's construction which employs the strong Kleene 3-valued logic.

The idea behind Kripke's construction is that the paradoxical sentences are defective, in much the way that sentences that contain denotationless proper names and sentences that contain category mistakes have been thought to be defective. Unlike semantically well-formed sentences, these defective sentences are neither true nor false. One uses a 3-valued logic to describe how these defective sentences interact with normal sentences.

Given a countable first-order language $\mathcal{L}$ and an acceptable structure $\mathfrak{A}$ for $\mathcal{L}$, we form the language $\mathcal{L}^+$ by adjoining the single new unary predicate '*Tr*' to $\mathcal{L}$. We expand $\mathfrak{A}$ to a *classical model* $(\mathfrak{A},E)$ of $\mathcal{L}^+$ by picking a subset E of $|\mathfrak{A}|$, which is to be the extension of '*Tr*'. We get a *partial model* $(\mathfrak{A},(E,A))$ by picking two disjoint subsets E and A of $|\mathfrak{A}|$. The *extension* E is to consist of those things to which the predicate '*Tr*' definitely applies, while the *anti-extension* A is to consist of those things to which the predicate '*Tr*' definitely does not apply. There

[1] An historically important example of a method for handling truth-value gaps which is not amenable to Kripke's techniques is the 3-valued logic of Łukasiewitz [1920]. We shall encounter another example in chapter 8.