

the analogy between these two properties, the specifications I, II, and the extremal clause, on the one hand, and the Peano postulates for natural numbers, on the other hand. ([CLg], sec. 2E7.)

B. THEORIES

In this section a theory will be defined as a class of statements. We shall consider here the formulation of this definition, and consequences following from it that do not require any assumptions concerning the objects which the statements of the theory refer to.

1. **Theories in general.** We begin by postulating a certain nonvoid, definite class \mathfrak{C} of statements, which we call *elementary statements*. As explained in Sec. A5, this means that the question of whether a given U expression does or does not express a statement of \mathfrak{C} is definite. The statements of \mathfrak{C} are called elementary statements to distinguish them from other statements which we may form from them or about them in the U language; later on we shall call some of these latter statements "epistatements," but for the moment we do not need this term.

A *theory* (over \mathfrak{C}) is defined as a conceptual class of these elementary statements. Let \mathfrak{I} be such a theory. Then the elementary statements which belong to \mathfrak{I} we shall call the *elementary theorems* of \mathfrak{I} ; we also say that these elementary statements are *true* for \mathfrak{I} . Thus, given \mathfrak{I} , an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of \mathfrak{C} a certain subclass of true statements. We shall then say that the statements of \mathfrak{C} constitute the elementary statements for (or of) the theory \mathfrak{I} .†

The terminology which has just been used implies that the elementary statements are not such that their truth and falsity are known to us without reference to \mathfrak{I} . The U sentences which express them must therefore contain some undetermined constituents or parameters whose meaning is not fixed until \mathfrak{I} is defined. In other words, they are *formal statements*, and they stand, in this respect, in contrast to the *contentive statements* whose truth and falsity are known to us completely beforehand. Of course, one may argue that this is improper usage; that the elements of \mathfrak{C} are not statements until the meaning of these undetermined constituents is fixed; and that therefore we must postulate a separate \mathfrak{C} for each \mathfrak{I} . This is, however, a matter of usage of terms. There are two arguments in favor of the usage here adopted. In the first place, it is convenient, in that it enables us to speak of two or more theories with the same \mathfrak{C} . In the second place, it agrees with the ordinary usage of the word 'sentence'¹; for the English expression

he is a jackass

is certainly a sentence, and one which my readers must have heard, yet it is not possible to judge of it as true or false until it is embedded in a context which will tell us what 'he' stands for and in which particular sense the word 'jackass' is intended. Later on we shall consider ways in which these

† It is not excluded that we may have theories with different classes \mathfrak{C} .

¹ It will be recalled that 'sentence' and 'statement' may be identified.