Next, we modify $H$ to produce a Turing machine $H'$ with the structure shown in Figure 12.2. With the added states in Figure 12.2 we want to convey that the transitions between state $q_y$ and the new states $q_a$ and $q_b$ are to be made, regardless of the tape symbol, in such a way that the tape remains unchanged. The way this is done is straightforward. Comparing $H$ and $H'$ we see that, in situations where $H$ reaches $q_y$ and halts, the modified machine $H'$ will enter an infinite loop. Formally, the action of $H'$ is described by

\[ q_0 w_M w \vdash^* H' \omega, \]

if $M$ applied to $w$ halts, and

\[ q_0 w_M w \vdash^* H' y_1 y_n y_2, \]

if $M$ applied to $w$ does not halt.

From $H'$ we construct another Turing machine $\hat{H}$. This new machine takes as input $w_M$, copies it, and then behaves exactly like $H'$. Then the action of $\hat{H}$ is such that

\[ q_0 w_M \vdash^* \hat{H} q_0 w_M w_M \vdash^* \hat{H} \omega, \]

if $M$ applied to $w_M$ halts, and

\[ q_0 w_M \vdash^* \hat{H} q_0 w_M w_M \vdash^* \hat{H} y_1 y_n y_2, \]

if $M$ applied to $w_M$ does not halt.